11.3 First-Order Predicate Logic

Exercise 3.1

(a) $\forall x\ \text{male}(x) \Leftrightarrow \neg\text{female}(x)$
(b) $\forall x\ \forall y\ \exists z\ \text{father}(x, y) \Leftrightarrow \text{male}(x) \wedge \text{child}(y, x, z)$
(c) $\forall x\ \forall y\ \text{siblings}(x, y) \Leftrightarrow [(\exists z\ \text{father}(z, x) \wedge \text{father}(z, y)) \vee (\exists z\ \text{mother}(z, x) \wedge \text{mother}(z, y))]$
(d) $\forall x\ \forall y\ \forall z\ \text{parents}(x, y, z) \Leftrightarrow \text{father}(x, z) \wedge \text{mother}(y, z)$
(e) $\forall x\ \forall y\ \text{uncle}(x, y) \Leftrightarrow \exists z\ \exists u\ \text{child}(y, z, u) \wedge \text{siblings}(z, x) \wedge \text{male}(x)$
(f) $\forall x\ \forall y\ \text{ancestor}(x, y) \Leftrightarrow \exists z\ \text{child}(y, x, z) \vee \exists u\ \exists v\ \text{child}(u, x, v) \wedge \text{ancestor}(u, y))$

Exercise 3.2

(a) $\forall x\ \exists y\ \exists z\ \text{father}(y, x) \wedge \text{mother}(z, x)$
(b) $\exists x\ \exists y\ \text{child}(y, x, z)$
(c) $\forall x\ \text{bird}(x) \Rightarrow \text{flies}(x)$
(d) $\exists x\ \exists y\ \exists z\ \text{animal}(x) \wedge \text{animal}(y) \wedge \text{eats}(x, y) \wedge \text{eats}(y, z) \wedge \text{grain}(z)$
(e) $\forall x\ \text{animal}(x) \Rightarrow (\exists y\ (\text{eats}(x, y) \wedge (\text{plant}(y) \vee (\text{animal}(y) \wedge \exists z\ \text{plant}(z) \wedge \text{eats}(y, z) \wedge \text{much\_smaller}(y, x)))$

Exercise 3.3 $\forall x\ \forall y\ \exists z\ \ x = \text{father}(y) \Leftrightarrow \text{male}(x) \wedge \text{child}(y, x, z)$

Exercise 3.4

$\forall x\ \forall y\ \ x < y \vee y < x \vee x = y,$
$\forall x\ \forall y\ \ x < y \Rightarrow \neg y < x,$
$\forall x\ \forall y\ \forall z\ \ x < y \wedge y < z \Rightarrow x < z$

Exercise 3.5

(a) MGU: $x/f(z)$, $u/f(y)$, term: $p(f(z), f(y))$
(b) not unifiable
(c) MGU: $x/\cos y$, $z/4 - 7 \cdot \cos y$, term: $\cos y = 4 - 7 \cdot \cos y$
(d) not unifiable
(e) MGU: $u/f(g(w,w), g(g(w,w), g(w,w)), g(g(g(w,w), g(w,w)), g(g(w,w), g(w,w))))$
$x/g(w,w), y/g(g(w,w), g(w,w))\ z/g(g(g(w,w), g(w,w)),\ g(g(w,w), g(w,w)))$
term: $q(f(g(w,w),\ g(g(w,w), g(w,w)),\ g(g(g(w,w), g(w,w)), g(g(w,w), g(w,w)))),$
$f(g(w,w),\ g(g(w,w), g(w,w)),\ g(g(g(w,w), g(w,w)), g(g(w,w),\ g(w,w)))))$

Exercise 3.7

(a) Let the unsatisfiable formula $p(x) \wedge \neg p(x) \wedge r(x)$ be given. We choose the clause $r(x)$ as the SOS, so no contradiction can be derived.
(b) If the SOS is already unsatisfiable, then no contradiction can be derived. If not, then resolution steps between clauses from SOS and $(KB \wedge \neg Q)$\SOS are necessary.